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A Nonparametric Approach to Monitoring Latency in Fiber Optic Networks: The Mixed DEWMA-CUSUM Sign Chart

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Abstract: The demand for reliable and fast data connections continues to rise alongside advancements in information technology, making fiber-optic networks the primary solution. However, maintaining network performance poses significant challenges, particularly in detecting latency anomalies caused by various factors. Conventional statistical monitoring approaches often rely on the assumption of normal distribution, which does not always align with characteristic of real-world network data. This study proposes the Mixed DEWMA-CUSUM Sign (MDCS) control chart, a nonparametric statistical method that combines DEWMA's sensitivity to small changes and CUSUM's efficiency in detecting process shifts. By employing a Sign-based approach, this method addresses the limitations of parametric techniques, ensuring more reliable performance in monitoring latency anomalies. The findings indicate that the MDCS method not only significantly improves anomaly detection but also contributes to the development of statistical methods for monitoring data with non-normal distributions, thereby enhancing overall network reliability.

Keywords: latency, fiber-optic network monitoring, mixed DEWMA-CUSUM sign chart, nonparametric statistical process control, data non-normal distribution

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1. INTRODUCTION

The increasing demand for fast and reliable data communication services has become increasingly crucial alongside the rapid development of information technology. Applications such as video streaming, social media, cloud storage, and gaming have now become integral parts of daily life [1]. A stable internet connection is also essential for business sectors that rely on reliable networks to enhance operational efficiency. However, challenges such as high latency, low throughput, and suboptimal bandwidth utilization remain major obstacles in network management [2][3].

Fiber-optic technology has emerged as a superior solution to meet the high data transmission needs, as it enables high-speed data transfer, large capacity, and resistance to electromagnetic interference [4][5]. Nevertheless, network disruptions such as increased latency can still occur due to factors like weather conditions, hardware, or software issues [6][7]. Anomalies such as increased latency can degrade service quality and user experience, making early detection a critical step in ensuring optimal network performance [8][9].

Statistical Process Control (SPC) offers various methods to systematically monitor changes in data. Among these methods, Exponentially Weighted Moving Average (EWMA), Double EWMA (DEWMA), and Cumulative Sum (CUSUM) are widely used techniques for detecting small yet significant changes [10]. DEWMA is known for its high sensitivity to subtle changes compared to conventional EWMA methods, while CUSUM excels in rapidly detecting minor changes [11]. Combining these two methods provides a more robust approach to monitoring dynamic data, such as latency in communication networks.

In practice, the assumption of normal distribution is often unmet, particularly in real-world data such as latency, which frequently exhibits unknown or non-normal distributions. To address this limitation, nonparametric SPC methods offer a more flexible solution [12]. The Mixed DEWMA-CUSUM Sign Chart is a nonparametric approach that uses the sign test to detect small changes in data. This method does not require distributional assumptions, is resistant to outliers, and is sensitive to minor changes, making it highly suitable for monitoring non-ideal data [13].

Previous research has shown that DEWMA and CUSUM each have their advantages in anomaly detection. For example, DEWMA demonstrates high sensitivity to small changes [14], while CUSUM is effective in reducing false alarm rates [15]. However, no studies have explicitly combined these two methods into an approach more suited for non-normally distributed data. The Mixed DEWMA-CUSUM Sign Chart offers a novel solution with high sensitivity for detecting various types of anomalies in non-normal data. This research is expected to contribute to the advancement of statistical science and the development of more reliable fiber-optic-based communication network monitoring systems.

By integrating DEWMA and CUSUM within a nonparametric framework, this study provides a comprehensive monitoring tool to address the complexities of real-world data. The proposed method is expected to enhance anomaly detection capabilities, strengthen network reliability, and support the adoption of advanced statistical techniques in monitoring dynamic systems.

2. BINOMIAL DATA DISTRIBUTION

The binomial distribution is used to describe the distribution of data from a series of trials conducted *n* times, where each trial is a repetition of a Bernoulli trial. In each repetition, there are only two possible outcomes: success or failure [16]. The binomial distribution has the following characteristics:

- Each trial has only two possible outcomes, such as *yes-no* or *success-failure*.
- The probability of an event remains constant and does not change for each trial.
- Each trial has only two possible results: success with probability p and failure with probability $q = 1 - p$.
- The trials are independent, meaning that the outcome of one trial does not affect or is not affected by the outcomes of other trials.
- The number of trials, which represents the binomial experiment component, must be fixed.

The binomial distribution formula is given by:

$$P(x) = \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

where:

- x : number of successes in n trials
- p : probability of success ($0 \le p \le 1$)
- n : number of trials
- q : probability of failure ($q = 1 - p$)

3. DEWMA *SIGN*

Based on the research conducted by Chao et al. [17], selecting an appropriate control chart is crucial for monitoring process changes. Specifically, when the quality characteristics of a process are unknown or the normality assumption is not met, nonparametric control charts become a suitable solution. The EWMA Sign control chart, which is nonparametric, was the first to be developed as a method for detecting deviations from the process target. This method was also designed to address the limitations of the Shewhart chart in identifying small-scale process changes.

Let X be a characteristic quality of a process with target value T. For instance, let $Y = X - T$

and $p = P(Y > 0)$ which represents the "process proportion". A value of $p = 0.5$ indicates that the process is in control. However, when $p = p_1 \neq 0.5$, it means the process is out of control, indicating that the process has shifted or changed due to a cause that can be identified, known as an "assignable cause". This refers to deviations or changes caused by external or abnormal factors, which are different from the usual random variation that occurs in the process. Suppose a random sample of size n is collected at time t, $X_{it}, i = 1,2,\ldots,n$, and $t = 1,2,\ldots$ to monitor irrelevance from the process target. It is then defined as

$$Y_{it} = X_{it} - T \text{ dan } I_{it} = \begin{cases} 1, & jika\ Y_{it} > 0 \\ 0, & sebaliknya \end{cases} \text{ untuk } i = 1,2,\ldots,n, \tag{12}$$

Now, if N_t represents the total count of I_{it}, then $N_t = \sum_{i=1}^{n} I_{it}$ it follows a binomial distribution with parameters $(n, 0.5)$ for a process that is in control.

A new control chart scheme called the Nonparametric Double EWMA (DEWMA) Sign chart is proposed to monitor the quality characteristic of a process whose distribution is unknown or not clearly identified. The statistical monitoring, denoted as DE_t, smooths the recorded numbers N_t, sequentially using EWMA, Z_t, as follows:

$$DE_t = \lambda_2 Z_t + (1-\lambda_2) DE_{t-1}, 0 < \lambda_2 \leq 1 \tag{13}$$

$$Z_t = \lambda_2 \sum_{j=1}^{t} \frac{1 - [(1-\lambda_2)/(1-\lambda_1)]^{t-j+1}}{1 - (1-\lambda_2)/(1-\lambda_1)} (1-\lambda_1)^{t-j} N_j \tag{14}$$

with the initial value $DE_0 = T = \frac{n}{2}$. Then, subtitute equation (14) into equation (13). DEWMA equation is

$$\begin{aligned} DE_t = {} & \lambda_1 \lambda_2 \sum_{j=1}^{t} \frac{1 - [(1-\lambda_2)/(1-\lambda_1)]^{t-j+1}}{1 - (1-\lambda_2)/(1-\lambda_1)} (1-\lambda_1)^{t-j} N_j \\ & + \left((1-\lambda_1)\lambda_2 \frac{(1-\lambda_1)^t - (1-\lambda_2)^t}{\lambda_2 - \lambda_1} + (1-\lambda_2)^t \right) T \end{aligned} \tag{15}$$

The average is $E(DE_t) = \frac{n}{2}$, and as $t \to \infty$, the variance is as follows:

$$Var(DE_t) = \frac{n}{4} . \left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} \right)^2 . \left\{ \frac{(1-\lambda_1)^2}{1-(1-\lambda_1)^2} - 2 \frac{(1-\lambda_1)(1-\lambda_2)}{1-(1-\lambda_1)(1-\lambda_2)} + \frac{(1-\lambda_2)^2}{1-(1-\lambda_2)^2} \right\}$$

Where the asymptotic variance is used to simplify the nonparametric DEWMA Sign control chart and to compare it with the EWMA scheme. Assuming that L represents the control limit width, the nonparametric DEWMA Sign control chart can be written as:

$$UCL = \frac{n}{2} + L\sqrt{Var(DE_t)}$$

$$CL = \frac{n}{2}$$

$$LCL = \frac{n}{2} - L\sqrt{Var(DE_t)}$$

As the process shifts, appropriate actions must be taken whenever DE_t falls outside the control

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limits.

Specifically, when $\lambda_1 = \lambda_2 = \lambda$ in equation (14) and $0 < \lambda \leq 1$, the nonparametric DEWMA equation can be rewritten as:

$$DE_t = \lambda^2 . \sum_{j=1}^{t} (t - j + 1)(1 - \lambda)^{t-j} N_j + (1 - \lambda)^t (1 + t\lambda) T \qquad (16)$$

The mean in this case is $E(DE_t) = \frac{n}{2}$, and as $t \to \infty$, the variance is

$$Var(DE_t) = \frac{n}{4} . \lambda^4 . \frac{1 + (1 - \lambda)^2}{[1 - (1 - \lambda)^2]^3}$$

In this case, the index i is not used, and only $t = 1,2, \ldots, n$. This is because the study involves only one variable, namely Pingtime Latency, and other factors contributing to network delay are not taken into account.

3. CUSUM *SIGN*

The CUSUM sign chart is a statistical tool used to monitor changes in the location of an industrial process without assuming any specific distribution of the data. This approach utilizes the sign of the data to detect shifts in the process median or central location. the primary advantage of this method is its distribution-free nature, making it suitable for various types of data without requiring specific distributional assumptions.

One significant reference discussing this topic is the article by Zameer et al., titled "A Distribution-free Adaptive CUSUM-Sign Chart for Monitoring Shifts in the Location of Unknown Industrial Process"[18]. This article introduces an adaptive, distribution-free CUSUM Sign Chart for monitoring process locations, providing a more flexible and effective approach for detecting location shifts without relying on specific data distribution assumptions.

The definition of a sign is as follows:

if $X_i > \theta_0$, then $S_i = +1$

if $X_i \leq \theta_0$, then $S_i = 0$

So that,

$$S_i = \begin{cases} +1, & if\ X_i > \theta_0 \\ 0, & if\ X_i \leq \theta_0 \end{cases}$$

CUSUM *Sign* Formula

$$C_i^+ = max(0, C_{i-1}^+ + sign(X_i - \theta_0) - k)$$
$$C_i^- = min(0, C_{i-1}^- + sign(X_i - \theta_0) + k)$$

with:

k : sensitivity factor to determine tolerance for changes

$C_0^+ = 0$ and $C_0^- = 0$ are the initial values of the CUSUM Sign

h : control limit

4. *MIXED DEWMA-CUSUM SIGN* (MDCS)

The mixed DEWMA-CUSUM Sign method combines the strengths of two different statistical techniques, namely DEWMA and CUSUM. This method is capable of providing more sensitive detection of small changes in network latency values.

The use of the Sign approach in CUSUM offers a simpler yet effective method for monitoring shifts in the data. The MDCS method is expected to optimize monitoring in fiber-optic networks, enhance anomaly detection, and maintain network quality and performance in stable conditions.

The formula for MDCS is as follows, with $C_0^+ = 0$ dan $C_0^- = 0$ as the initial values of the CUSUM $Sign$. DE_t represents the value of DEWMA $Sign$, and θ_0 is the target value used as a reference to determine whether the data is in control or out of control.

$$\text{If } X_i > \theta_0, \text{ then } S_i = +1$$
$$\text{If } X_i \leq \theta_0, \text{ then } S_i = 0$$

Thus:

$$S_i = \begin{cases} +1, & jika\ X_i > \theta_0 \\ 0, & jika\ X_i \leq \theta_0 \end{cases}$$

Hence, the Mixed DEWMA-CUSUM Sign (MDCS) is defined as follows:

$$C_i^+ = max(0, C_{i-1}^+ + sign(DE_t - \theta_0) - k)$$
$$C_i^- = min(0, C_{i-1}^- + sign(DE_t - \theta_0) + k)$$

where DE_t is given as follows, due to the values of λ used is the same :

$$DE_t = \lambda^2. \sum_{j=1}^{t} (t - j + 1)(1 - \lambda)^{t-j} N_j + (1 - \lambda)^t (1 + t\lambda) T$$

By assuming that L represents the control limit width, the formula of upper and lower control limit of nonparametric DEWMA Sign control chart can be written as:

$$UCL = \frac{n}{2} + L\sqrt{Var(DE_t)}$$

$$CL = \frac{n}{2}$$

$$LCL = \frac{n}{2} - L\sqrt{Var(DE_t)}$$

The mean in this case is $E(DE_t) = \frac{n}{2}$, and as $t \to \infty$, the variance is

$$Var(DE_t) = \frac{n}{4}. \lambda^4. \frac{1+(1-\lambda)^2}{[1-(1-\lambda)^2]^3}$$

5. MONTE CARLO ARL

According to The Average Run Length (ARL) is defined as the average number of sample points that need to be plotted on a control chart before one of the points indicates a statistically out-of-control condition. It serves as a measure of the performance of a control chart. The smaller the ARL value on a control chart, the faster the chart detects the out-of-control condition.

Monte Carlo simulation is an analytical technique that uses random data to generate probability statistics, helping to understand the impact of uncertainty. this technique operates as a simple program that generates random data based on a specific distribution for simulation process. The ARL value can be calculated using this method. the following equation can be used to estimate the ARL value through Monte Carlo simulation

$$ARL = \frac{\sum_{t=1}^{N} RL_t}{N}$$

with:

ARL : The average number of sample points before an out-of-control condition occurs.

RL_t : The number of sample points before an out-of-control condition occurs in the t-th simulation

N : Number of simulation repetitions

5. Main Results

DEWMA Sign

The results of the DEWMA Sign analysis with binomially distributed data from fiber-optic-based network latency for Company 1 (C1), Company 2 (C2), dan Company 3 (C3), using the parameters $\lambda = 0.5,\ L = 3,\ and\ h = 4$, as follows:

Table 1. OOC and ARL DEWMA Sign with λ=0.5

C1		C2		C3	
OOC	ARL	OOC	ARL	OOC	ARL
14	512,54	40	512,54	7	512,54

The table shows the results of the DEWMA sign analysis with a weight of λ = 0.5 applied to three companies. the number of out-of-control data point is 14 at C1, 40 at C2, and 7 OOC at C3, respectively, with the same ARL value, 512.54. These results indicate that P2 has the highest level of anomalies, while C3 is the most stable. The consistent ARL across all companies demonstrates the method's reliability in detecting anomalies at the given λ weight. The corresponding plot is as follows:

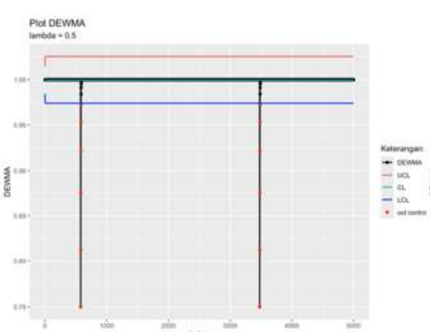

Figure 1. DEWMA Sign C1

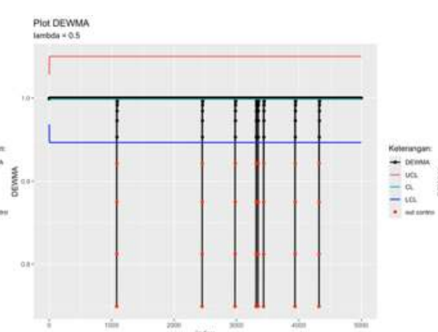

Figure 2. DEWMA Sign C2

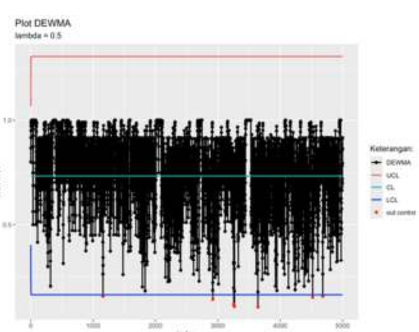

Figure 3. DEWMA Sign C3

Table 2 presents the results of the DEWMA Sign analysis applied to three companies, namely Company 1 (P1), Company 2 (P2), and Company 3 (P3), with varying parameter values of λ (ranging from 0.1 to 0.9). The number of out-of-control (OOC) data points and the ARL

(Average Run Length) values vary depending on the λ parameter used. As the value of λ increases, the number of OOC data points in all companies tends to decrease, and the ARL also shows a significant reduction.

For instance, at λ = 0.1, the ARL reaches 1520.641, with the highest number of OOC data points recorded: 66 (P1), 179 (P2), and 192 (P3). Conversely, at λ = 0.5, the ARL decreases to 512.54, with the lowest number of OOC data points: 14 (P1), 40 (P2), and 7 (P3). These findings indicate that increasing the value of λ enhances the responsiveness and effectiveness of the DEWMA Sign method in detecting minor changes during monitoring, particularly in reducing the number of data points classified as out of control.

Table 2. OOC and ARL DEWMA Sign λ=0.1-0.5

λ	ARL	P1	P2	P3
		OOC	OOC	OOC
0.1	1520.641	66	179	192
0.2	1251.153	36	112	3
0.3	849.794	24	72	5
0.4	630.266	18	56	4
0.5	512.54	14	40	7
0.6	454.933	10	32	3
0.7	399.644	8	24	0
0.8	379.836	6	16	0
0.9	379.836	4	16	0

A higher ARL indicates that the system takes longer to detect OOC conditions, resulting in lower sensitivity. Conversely, a larger λ value accelerates anomaly detection with a lower ARL but poses a higher risk of false alarms, especially for small-scale anomalies.

For fiber-optic-based networks with moderate fluctuation levels, λ values between 0.4 and 0.6 appear to provide the best balance between sensitivity and reliability, making them suitable as optimal parameters.

CUSUM Sign

The results of the CUSUM Sign analysis using binomially distributed data from fiber-optic-based network latency for Company 1, Company 2, and Company 3, with parameters $\lambda = 0.5$, $L = 3, h = 4$, and $\theta = 0.25$, are as follows:

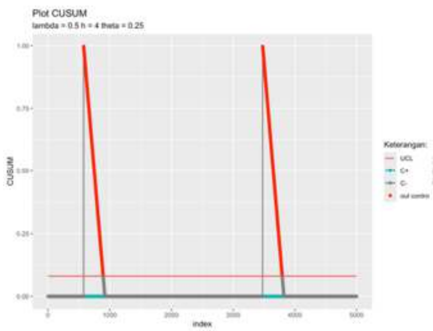

Figure 4. CUSUM Sign C1

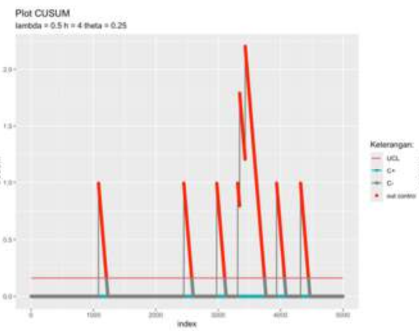

Figure 5. CUSUM Sign C2

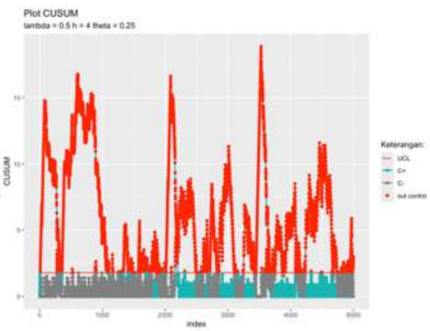

Figure 6. CUSUM Sign C3

In the CUSUM Sign analysis, the number of data points detected as out of control in each company is as follows: 634 data points for Company 1 (C1), 1,065 data points for Company 2 (C2), and 3,991 data points for Company 3 (C3). Using the Monte Carlo approach, the Average Run Length (ARL) obtained is 20.261.

The ARL value indicates that the CUSUM Sign method tends to be more stable and less sensitive to minor fluctuations in the data. This is crucial in reducing the number of false alarms; however, it may delay detection time if anomalies occur suddenly.

Mixed DEWMA-CUSUM Sign

The results of the Mixed DEWMA-CUSUM Sign analysis using binomially distributed data from fiber-optic-based network latency in Company 1, Company 2, and Company 3, with parameters $\lambda = 0.5,\ L = 3, h = 4$, are as follows:

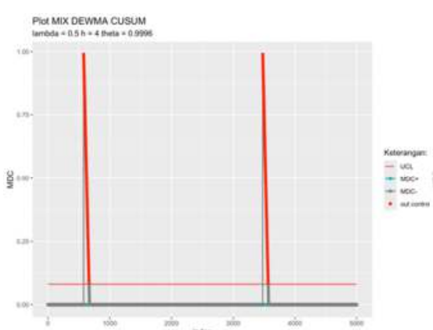

Figure 7. MDC Sign P1

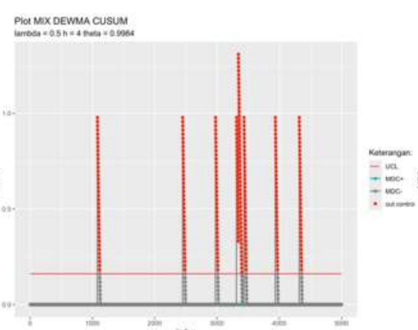

Figure 8. MDC Sign P2

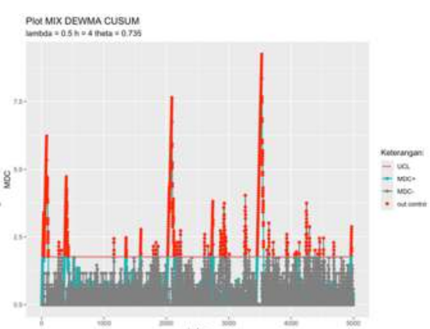

Figure 9. MDC Sign P3

By applying the MDCS method, the analysis results indicate that the number of *out-of-control* data points is 236 for Company 1, 421 for Company 2, and 586 for Company 3. Using the Monte Carlo approach, the obtained *Average Run Length* (ARL) is 17.941.

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The following table presents a comparison of the results between the DEWMAS, CUSUMS, and MDCS methods in detecting anomalies in network latency monitoring:

Table 3. DEWMAS, CUSUMS, and MDCS, λ=0.5

P	DEWMAS		CUSUMS		MDCS	
	OOC	ARL	OOC	ARL	OOC	ARL
P1	14	512.54	634	20.261	236	17.941
P2	40		1065		421	
P3	7		3991		586	

Overall, the MDCS method demonstrates that selecting an appropriate monitoring strategy and optimizing parameter settings are crucial for ensuring effective anomaly detection. The use of parameter λ = 0.5 provides an optimal balance between sensitivity and reliability in detecting anomalies in fiber-optic-based network latency monitoring. Meanwhile, the *Mixed DEWMA-CUSUM Sign* method showed the smallest ARL Value among other method, which MDCS is faster to detects the out-of-control condition, and showed that MDCS serves as a flexible and responsive tool for identifying both minor and significant changes in network performance.

CONCLUSION

In this study, we have explored the effectiveness of three anomaly detection methods, DEWMA Sign, CUSUM Sign, and the Mixed DEWMA-CUSUM Sign (MDCS) for monitoring network latency in fiber-optic-based systems across multiple companies. The results highlight the strengths and limitations of each method, particularly in relation to their ability to detect out-of-control conditions and their response times.

The DEWMA Sign method, with its parameter λ = 0.5, demonstrated a solid performance in detecting anomalies across all three companies. It exhibited a balance between sensitivity and reliability, with the number of out-of-control data points decreasing as the sensitivity factor λ increased. However, this also resulted in a decrease in the Average Run Length (ARL), which suggests faster detection but also a higher risk of false alarms for small-scale anomalies. For the optimal balance between sensitivity and reliability, a λ value between 0.4 and 0.6 proved to be the most effective.

The CUSUM Sign method, in contrast, showed a higher ARL and detected more out-of-control data points, especially for Company 3. This indicates that CUSUM Sign is more stable and less sensitive to minor fluctuations, reducing the likelihood of false alarms. However, this comes at the cost of delayed anomaly detection, particularly for sudden shifts in the data.

In this research, the Mixed DEWMA-CUSUM Sign method (MDCS) emerged as the most flexible and responsive tool for network performance monitoring. By combining the strengths of both DEWMA and CUSUM, the MDCS method provided a better balance between sensitivity and reliability, as evidenced by the smallest ARL values observed. The results revealed that MDCS is faster at detecting out-of-control conditions and is capable of identifying both minor and significant changes in latency more effectively than either DEWMA or CUSUM alone.

In summary, the Mixed DEWMA-CUSUM Sign method offers a promising approach for real-time anomaly detection in fiber-optic networks, particularly when optimized with appropriate parameters. Its ability to quickly detect changes while maintaining a low ARL makes it a powerful tool for network monitoring, capable of handling both subtle and substantial shifts in performance.

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